What does an AC voltmeter measure?

By Louis Dudzik 11/20/05 Updated 7/12/09

The purpose of the article is to determine what method a typical, low-cost, AC voltmeter uses to approximate the AC RMS value of a voltage waveform.

Why use RMS?

There needs to be a way of rating voltage waveforms. If it's a constant DC voltage, it is simple. It is the DC voltage value. If the voltage waveform is not a constant DC value, it is more difficult.

Often, the energy that a voltage waveform can produce is of more interest than the actual voltage itself. Because of this, a method of rating voltage waveforms is needed such that the rating could be used to indicate the average energy the voltage waveform would produce in a resistive load. In other words, if there are two totally different voltage waveforms, but they are both able to cause the same average energy to be dissipated in a resistive load, they should have the same rated value.

At first, it would seem that the average voltage would be a good choice. However, two waveforms with the same average voltage may not necessarily produce the same average power. This is obvious in the case of an AC voltage waveform where the average may be zero, but it may not be obvious in the case of DC waveforms that fluctuate. If a DC voltage is comprised of a fluctuating waveform, it may produce more power than the average of the waveform would indicate. Therefore, average voltage is not a good choice for finding the power-producing potential of a voltage waveform.

For example, consider a constant DC waveform at 1 volt as a reference waveform. In addition, consider a pulsed DC waveform (square wave) with a 50% duty cycle with a peak of 2 volts. Both have the same average voltage, (1 volt), but the square wave delivers twice the power of the reference waveform. Why? It is because doubling the voltage also doubles the current. The power is quadrupled, when compared to the reference waveform, during the pulses. Doubling the voltage for half the time gives the same average voltage of the reference voltage. Quadrupling the power for half the time gives twice the average power of the reference voltage.

Peak-voltage values are also a poor choice for rating voltages. Peak-voltage is almost meaningless when trying to rate the average power produced by a voltage waveform.

The RMS value of a waveform (as defined below) achieves the desired characteristic for a rating method. Two voltage waveforms that have the same total RMS value will produce the same average energy-dissipation in a resistive load. The "total" RMS value refers to the RMS value for an entire waveform, not just the AC component. The waveforms may actually contain any combination of AC and DC components.

In order to get more information about a waveform, it is usually broken up into its AC component and DC component. The DC component is assumed to be a steady DC value. For steady DC voltages, the average voltage is equal to the RMS voltage. The average voltage is all that is necessary to define the DC component of a waveform. The AC component however, by its nature, is not steady. Therefore, it is desired for the AC component to be measured in RMS values.

RMS defined:

Given a function f(t),

$$F_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} f^2(t)dt}$$

Frms is the root-mean-square (RMS) value of the function f(t), where t is time.

In other words, to find Frms, the function f(t) is, first, squared.

Next, the average of the squared function is found by calculating the definite integral over one period T. The integral is then divided by T (the length of one period). This is the average (mean) of the squared function.

Finally, the square root of this average is Frms.

If f(t) is a voltage function, we can use v(t) to represent the voltage. Then Vrms would represent the RMS value of v(t). The general formula, then, becomes:

$$V_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)dt}$$

This is for any continuous AC or DC voltage function.
The RMS value is just a single value for a repeating function (or waveform). For a non-repeating function, the RMS value may change continuously and would, itself, be a function of time.

Note: The RMS value of a voltage waveform is the same whether it is rectified or not.
This is not the case for the average value of a waveform.

What Does a Voltmeter Measure?

A voltmeter is used to measure DC voltages and AC voltages.

The DC voltmeter is not measuring RMS voltage. When one is measuring the DC component of a waveform, one would usually like to ignore the AC component. The DC component is the average voltage of the waveform. The average voltage is easier for a meter to measure than RMS, so it is fortunate that one usually only wants the DC average value, and not the RMS value. When a voltage waveform is a constant value (i.e. pure DC), the DC average voltage and the total RMS voltage are the same.

However, there are instances when the total RMS value is very different from the DC average. Any time there is a repeating change in voltage, the DC average is likely to be different from the RMS value. The significance one must realize is that the energy delivered by such a fluctuating voltage is more than what the DC average value would indicate. Therefore, it is important to know if a significant AC component exists in the waveform.

To measure the AC component, one would use an AC voltmeter. However, most low-cost voltmeters also won't measure RMS when measuring AC. In this case, however, it is not because it is not desired, but because it is not easy to do. Very expensive voltmeters are often made to measure "true RMS", because that is what is desired.

Low-cost voltmeters approximate the RMS value. To approximate the RMS value for a sine wave, one could simply find the peak value of the sine wave and multiply it by .707. This is because the RMS value of a perfect AC sine wave is .707 times the peak. This would be a very poor method of approximating any other type of waveform, though. The chances of any random waveform having an RMS value near .707 times its peak value is very slim. Also, any "noise" could throw the approximation way off.

A better method of approximating AC RMS value would be to make use of what the meter already has: an average-voltage-detector. The meter already has the circuit for finding the average of a waveform, and noise does not affect the average reading nearly as much as it affects the peak reading.

To use the average-voltage-detector to approximate AC RMS would not be very difficult. First, a coupling capacitor would eliminate the DC average (or DC offset) from the waveform. Next, an "ideal rectifier" would rectify the remaining AC portion. (The "ideal rectifier" rectifies the waveform without any voltage drops associated with using diodes. It is built with op-amps instead.) This AC waveform, rectified, will then go to the average-voltage-detector. The result of that is then multiplied by 1.1 in order get the result to equal the RMS value for a sine wave. The value of 1.1 comes from dividing the RMS value for a sine wave by the rectified-average of a sine wave. The RMS value for a sine wave is approximately .707 times the peak value. The rectified-average of a sine wave is approximately .637 times the peak value. (These values are derived later.)

$$\frac{.707}{.637} \cong 1.1$$

Once again, this method would be exact when trying to obtain the RMS value of a pure sine wave, but would still only be an approximation of RMS for other waveforms. However, the approximation is much closer than with the peak-times-.707 method (for non-sine waves) and would be less affected by noise.

The hypothesis that the AC-rectified-average method, suggested above, is the method used by most low-cost voltmeters for approximating RMS voltage values for AC waveforms is what will be tested here.
Update 7/12/09: Some use another method. See Update on page 8.

Different Voltage Measurement Types

At this point, several different types of rating methods have been mentioned for measuring voltage. Here is a brief list with a short description of each. Some of these terms are being defined in the context of this discussion. The terms may mean something else in another context.

Average Voltage, DC Average, DC Voltage, DC Component, (DC Offset): These basically all mean the same thing when used in reference to a DC voltmeter. It is the average voltage of the entire waveform. It is found as the integral of the waveform divided by the interval of the integral.

This is what a typical DC voltmeter measures.

Sometimes “DC offset” doesn’t necessarily refer to the average voltage, though. Sometimes it is referring simply to some DC value by which a waveform is transposed. It must be understood if this is the case when using the term “DC offset”.

Total RMS Voltage, RMS Voltage: Formally, this is the RMS voltage value, as defined above, of the entire waveform. However, the term “RMS Voltage”, unfortunately, is often used to refer to the RMS value of the AC component of a waveform. It is important to be clear to avoid confusion.

DC RMS Voltage: This term is ambiguous and should be avoided. It could mean the RMS value of the DC component (which would be the same as the average voltage) or it could mean the RMS value of the entire waveform. These two values are not necessarily the same.

AC RMS Voltage, AC Voltage, RMS Voltage: This refers to the RMS voltage value, as defined above, of the AC component of a waveform. This is what expensive, “true RMS”, AC voltmeters measure and what a typical AC voltmeter approximates.

Formally, “RMS Voltage” is the RMS voltage value, as defined above, of the entire waveform. However, the term “RMS Voltage” is often used to refer to the RMS value of the AC component of a waveform. It is important to be clear to avoid confusion.

Rectified Average: This is the average voltage of an entire waveform after it has been rectified. (The DC average is not removed before the waveform is rectified.)

AC Rectified Average: This term refers to the average voltage of the rectified AC component of a waveform. (The DC average is removed before the waveform is rectified.)

This, multiplied by 1.1, is what a typical AC voltmeter uses to approximate the AC RMS voltage of a waveform is the hypothesis of this article.

NOTE: To get the total RMS value of the signal, one cannot simply add the DC average component and the AC RMS component. One must add the squares of the components then take the square root of the sum.

Voltage values for common waveforms:

These are general formulas for finding the 5 different types of voltage rating methods for common waveforms with various DC offsets (and varying duty cycles for square waves). The derivations for these formulas can be found in the appendices.

Sine Waves, Cosine Waves:

Sine waves and cosine waves have the same measurement formulas. To make the math easier in the derivations, cosines were used.

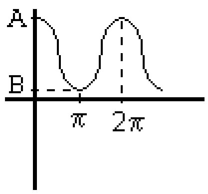

Fig. 1

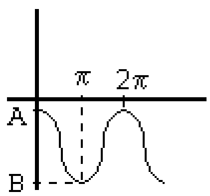

Fig. 2

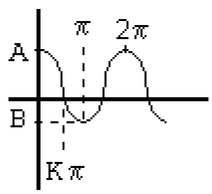

Fig. 3

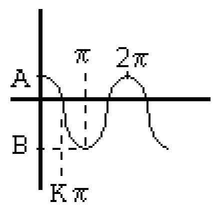

Fig. 4

v(t) is a cosine wave.
A is the upper peak voltage.
B is the lower peak voltage.
A is always greater than or equal to B.
K is a fraction. K only exists if V(t) crosses the horizontal axis.
K multiplied by π is where V(t) first crosses the horizontal axis.
In other words, K multiplied by π is where V(t) first changes from positive to negative.

Average Voltage = $\frac{(A+B)}{2}$

Total RMS Voltage = $\sqrt{\frac{(A-B)^2}{8}+\frac{(A+B)^2}{4}}$

E = A-Average Voltage
F = B-Average Voltage

AC RMS Voltage = $\frac{E}{\sqrt{2}}$

K = $\frac{1}{\pi}\arccos\left(\frac{B+A}{B-A}\right)$ (Where $\arccos\left(\frac{B+A}{B-A}\right)$ is in Radians.)

(Arcos is the inverse of the cosine function. Arcos is sometimes notated as $\cos^{-1}$.)

If $B \geq 0$, **Rectified Average Voltage** = Average Voltage
If $A \leq 0$, **Rectified Average Voltage** = -Average Voltage
For all other cases, **Rectified Average Voltage** = $\frac{(A-B)}{\pi}\sin(K\pi)+K(A+B)-\frac{(A+B)}{2}$

AC Rectified Average Voltage = $\frac{2E}{\pi}$

Sawtooth Waves, Triangle Waves:

Sawtooth waves and triangle waves have the same formulas due to symmetry. Half of a triangle wave is a sawtooth wave. Also, a reversed sawtooth wave has the same formulas and results as a forward sawtooth wave.

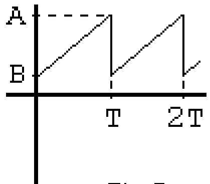

Fig.5

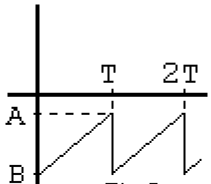

Fig.6

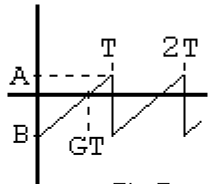

Fig.7

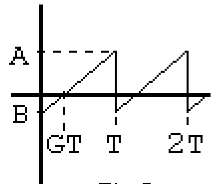

Fig.8

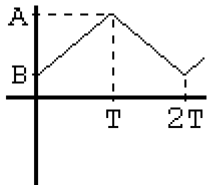

Fig. 9

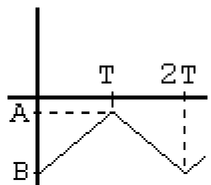

Fig.10

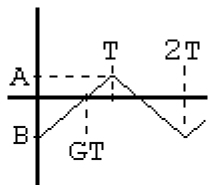

Fig.11

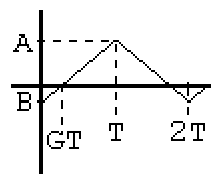

Fig.12

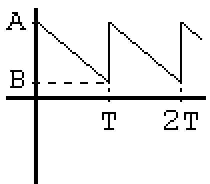

Fig.13

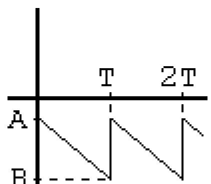

Fig.14

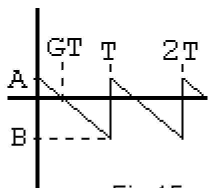

Fig.15

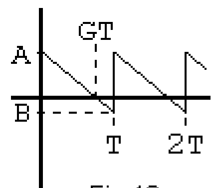

Fig.16

v(t) is a sawtooth wave or a triangle wave.
A is the upper peak voltage.
B is the lower peak voltage.
A is always greater than or equal to B.
T is the period of the sawtooth wave or ½ of the period of a triangle wave.
G is a fraction. G only exists if V(t) crosses the horizontal axis.
G multiplied by T is where V(t) first crosses the horizontal axis.
In other words, G multiplied by T is where V(t) first changes from positive to negative or negative to positive.

Average Voltage = $\frac{(A+B)}{2}$

Total RMS Voltage = $\sqrt{\frac{A^2+AB+B^2}{3}}$

E = A-Average Voltage
F = B-Average Voltage

AC RMS Voltage = $\frac{E}{\sqrt{3}}$

If $B \geq 0$, **Rectified Average Voltage** = Average Voltage
If $A \leq 0$, **Rectified Average Voltage** = -Average Voltage

For all other cases, **Rectified Average Voltage** = $\frac{A^2+B^2}{2A-2B}$

AC Rectified Average Voltage = $\frac{E}{2}$

Square Wave with Duty Cycle:

Square waves are simple until the duty cycle is altered from 50%. When the duty cycle is not at 50%, the DC average (or DC offset) is no longer at the center (vertically) of the waveform. For a square wave, the duty cycle is the percentage of time the waveform is at its most positive voltage.

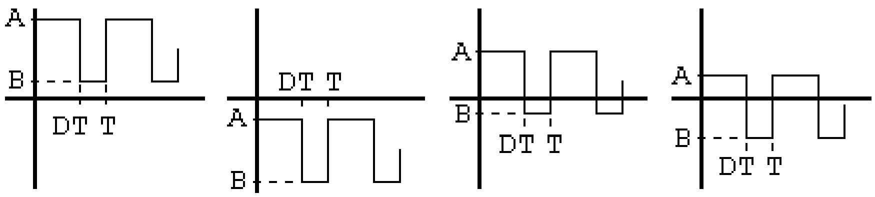

Fig.17 Fig.18 Fig.19 Fig. 20

v(t) is a square wave.
A is the upper peak voltage.
B is the lower peak voltage.
A is always greater than or equal to B.
T is the period of the square wave.
D is a fraction. D multiplied by 100% is the duty cycle.
D multiplied by T is where V(t) first transitions from A to B.

Average Voltage = $AD + B - BD$
It is important to note that this is not necessarily at the midpoint between A and B.

Total RMS Voltage = $\sqrt{A^2D + B^2 - B^2D}$

E = A-Average Voltage
F = B-Average Voltage

AC RMS Voltage = $\sqrt{E^2D + F^2 - F^2D}$

If $B \geq 0$, **Rectified Average Voltage** = Average Voltage
If $A \leq 0$, **Rectified Average Voltage** = -Average Voltage
For all other cases, **Rectified Average Voltage =** $AD - B + BD$

AC Rectified Average Voltage = $ED - F + FD$

Tests:

Since measuring peak values for a waveform is easy to do on an oscilloscope, the values for A and B can be measured directly. From those measurements, the other various types of voltage rating methods can be calculated. The results can be compared to the actual measurements of a multi-meter to determine what method the meter is using to rate a voltage waveform.

Test waveforms were created using a sine wave generator, triangle wave generator, square wave generator, and a circuit to add a variable DC offset to the waveform along with a voltage follower with variable gain. An oscilloscope and a Fluke 73 multi-meter measured the waveforms simultaneously. The oscilloscope was used to determine the peak values (A and B) and the duty-cycle only. The other values were calculated based on those measurements. Those values were compared to the actual measurements made on the Fluke.

Peak x .707 method uses the upper peak, after DC average is removed, for the reference.
Peak = A – DC average.

In almost every case, the measured value was closer to the AC-rectified-average x 1.1 than true RMS or peak x .707 . This clearly supports the hypothesis.

Update 7/12/09: Since the initial writing of this paper, a few ultra-low priced meters have become available. These are typically made in China and retail for less than ten dollars, and often less than five dollars. These meters employ a different approach to measuring AC voltage. It was discovered that they simply eliminate the negative portion of the voltage wave by means of a half-wave rectifier, take the average of the positive portion, and double it. That is, AC voltage = 2 times the average of the positive half of the voltage wave. This type of meter can easily be identified because it will not register any AC voltage if a battery is placed on the test leads with negative polarity, but will show twice the DC value of the battery voltage if the battery is placed on the test leads with positive polarity.

Test Data:

- The data shows the type of waveform tested and the frequency.
- A and B (in volts) are the upper peak and lower peak, respectively, as measured on the oscilloscope.
- The Duty-cycle (on square waves) was calculated from measurements made on the oscilloscope.
- All other calculated values were calculated using A, B, and Duty Cycle.
- DC-average (in volts) is the calculated DC component of the waveform.
- Fluke-measured-DC-average (in volts) is the Fluke's DC voltage measurement of the waveform. It should agree with the calculated DC-average.
- AC-RMS (in volts) is the calculated RMS voltage of the AC component of the waveform.
- Peak x .707 (in volts) is .707 times the calculated upper peak of the AC component. The upper peak of the AC component is simply A minus the DC average.
- AC-rectified-average x 1.1 (in volts) is the calculated, rectified average of the AC component multiplied by 1.1. According to the hypothesis, this is the value that the AC voltmeter is actually measuring in order to approximate the true RMS voltage of the AC component of the waveform.
- Fluke-measured-AC-voltage (in volts) is the Fluke's AC voltage measurement of the waveform. According to the hypothesis, it should agree with the calculated AC-rectified-average x 1.1 .

Test 1:
Sine Wave 30 Hz
A = 8.7
B = -4.1
DC average = 2.30
Fluke measured DC average = 2.23
AC RMS = 4.53
Peak x .707 = 4.53
AC rectified average x 1.1 = 4.53
Fluke measured AC voltage = 4.54

Test 2:
Sine Wave 30 Hz
A = 6.4
B = -6.4
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 4.53
Peak x .707 = 4.53
AC rectified average x 1.1 = 4.53
Fluke measured AC voltage = 4.54

Test 3:
Sine Wave 42 Hz
A = 2.6
B = -2.6
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 1.84
Peak x .707 = 1.84
AC rectified average x 1.1 = 1.84
Fluke measured AC voltage = 1.87

Test 4:
Sine Wave 60 Hz
A = 6.4
B = -6.4
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 4.53
Peak x .707 = 4.53
AC rectified average x 1.1 = 4.53
Fluke measured AC voltage = 4.52

Test 5:
Sine Wave 60 Hz
A = 7.6
B = 0.0
DC average = 3.80
Fluke measured DC average = 3.78
AC RMS = 2.69
Peak x .707 = 2.69
AC rectified average x 1.1 = 2.69
Fluke measured AC voltage = 2.69

Test 6:
Sine Wave 60 Hz
A = 8.7
B = -8.7
DC average = 0.00
Fluke measured DC average = .028
AC RMS = 6.15
Peak x .707 = 6.15
AC rectified average x 1.1 = 6.15
Fluke measured AC voltage = 6.19

Test 7:
Sine Wave 1100 Hz
A = 9.0
B = -9.0
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 6.36
Peak x .707 = 6.36
AC rectified average x 1.1 = 6.36
Fluke measured AC voltage = 6.72

Test 8:
Sine Wave 4900 Hz
A = 9.0
B = -9.0
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 6.36
Peak x .707 = 6.36
AC rectified average x 1.1 = 6.36
Fluke measured AC voltage = 6.36

Test 9:
Here it was noticed that at 5000 Hz, the lowest AC-voltage scale on the Fluke did not register properly. It was not determined if this behavior was isolated to the one meter being tested or if all Fluke 73 meters behave this way. The reading returned by the lowest scale did not agree with the reading returned by the other scales (1.409 versus 1.94). For this reason, the lowest scale's reading was ignored and the reading was taken using the next lowest scale.

Sine Wave 5000 Hz
A = 2.7
B = -2.7
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 1.91
Peak x .707 = 1.91
AC rectified average x 1.1 = 1.91
Fluke measured AC voltage = 1.94

Test 10:
Sawtooth/Triangle Wave 120 Hz
A = 8.3
B = -8.3
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 4.79
Peak x .707 = 5.87
AC rectified average x 1.1 = 4.61
Fluke measured AC voltage = 4.70

Test 11:
Sawtooth/Triangle Wave 120 Hz
A = 9.0
B = -3.5
DC average = 2.75
Fluke measured DC average = 2.80
AC RMS = 3.61
Peak x .707 = 4.42
AC rectified average x 1.1 = 3.47
Fluke measured AC voltage = 3.53

Test 12:
Sawtooth/Triangle Wave 240 Hz
A = 9.0
B = -3.4
DC average = 2.80
Fluke measured DC average = 2.81
AC RMS = 3.58
Peak x .707 = 4.38
AC rectified average x 1.1 = 3.44
Fluke measured AC voltage = 3.49

Test 13:
Sawtooth/Triangle Wave   240 Hz
A = 6.25
B = -6.25
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 3.61
Peak x .707 = 4.42
AC rectified average x 1.1 = 3.47
Fluke measured AC voltage = 3.48

Test 14:
Sawtooth/Triangle Wave   500 Hz
A = 6.2
B = -6.2
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 3.58
Peak x .707 = 4.38
AC rectified average x 1.1 = 3.44
Fluke measured AC voltage = 3.48

Test 15:
Sawtooth/Triangle Wave   500 Hz
A = 8.5
B = -4.0
DC average = 2.25
Fluke measured DC average = 2.24
AC RMS = 3.61
Peak x .707 = 4.42
AC rectified average x 1.1 = 3.47
Fluke measured AC voltage = 3.50

Test 16:
Sawtooth/Triangle Wave   860 Hz
A = 10.2
B = -3.8
DC average = 3.20
Fluke measured DC average = 3.23
AC RMS = 4.04
Peak x .707 = 4.95
AC rectified average x 1.1 = 3.89
Fluke measured AC voltage = 3.91

Test 17:
Sawtooth/Triangle Wave   1260 Hz
A = 9.1
B = -9.1
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 5.25
Peak x .707 = 6.43
AC rectified average x 1.1 = 5.05
Fluke measured AC voltage = 4.91

Test 18:
Sawtooth/Triangle Wave   1260 Hz
A = 1.7
B = -5.7
DC average = -2.00
Fluke measured DC average = -2.00
AC RMS = 2.14
Peak x .707 = 2.62
AC rectified average x 1.1 = 2.05
Fluke measured AC voltage = 2.04

Test 19:
Square Wave   500 Hz
A = 5.5
B = -5.5
Duty Cycle = 50 %
DC average = 0.00
Fluke measured DC average = 0.002
AC RMS = 5.50
Peak x .707 = 3.89
AC rectified average x 1.1 = 6.11
Fluke measured AC voltage = 6.10

Test 20:
Square Wave   500 Hz
A = 8.3
B = -2.0
Duty Cycle = 50 %
DC average = 3.15
Fluke measured DC average = 3.11
AC RMS = 5.15
Peak x .707 = 3.64
AC rectified average x 1.1 = 5.72
Fluke measured AC voltage = 5.69

Test 21:
Square Wave   3000 Hz
A = 8.3
B = -1.7
Duty Cycle = 50 %
DC average = 3.30
Fluke measured DC average = 3.11
AC RMS = 5.00
Peak x .707 = 3.54
AC rectified average x 1.1 = 5.55
Fluke measured AC voltage = 5.37

Test 22:
Square Wave   3000 Hz
A = 5.0
B = -5.0
Duty Cycle = 50 %
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 5.00
Peak x .707 = 3.54
AC rectified average x 1.1 = 5.55
Fluke measured AC voltage = 5.36

Test 23:
Square Wave   30 Hz
A = 4.6
B = -10.0
Duty Cycle = 50 %
DC average = -2.70
Fluke measured DC average = -2.56
AC RMS = 7.30
Peak x .707 = 5.16
AC rectified average x 1.1 = 8.11
Fluke measured AC voltage = 7.98

Test 24:
Square Wave   30 Hz
A = 7.3
B = -7.3
Duty Cycle = 50 %
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 7.30
Peak x .707 = 5.16
AC rectified average x 1.1 = 8.11
Fluke measured AC voltage = 7.98

Test 25:
Square Wave   60 Hz
A = 7.3
B = -7.3
Duty Cycle = 50 %
DC average = 0.00
Fluke measured DC average = 0.00
AC RMS = 7.30
Peak x .707 = 5.16
AC rectified average x 1.1 = 8.11
Fluke measured AC voltage = 8.13

Test 26:
Square Wave   60 Hz
A = 4.5
B = -9.9
Duty Cycle = 50 %
DC average = -2.70
Fluke measured DC average = -2.56
AC RMS = 7.20
Peak x .707 = 5.09
AC rectified average x 1.1 = 8.00
Fluke measured AC voltage = 8.04

Test 27:
Square Wave   100 Hz
A = 1.9
B = -8.6
Duty Cycle = 91 %
DC average = 0.955
Fluke measured DC average = 0.911
AC RMS = 3.00
Peak x .707 = 0.67
AC rectified average x 1.1 = 1.91
Fluke measured AC voltage = 1.92

Test 28:
Square Wave   60 Hz
A = -1.8
B = -12.2
Duty Cycle = 78.1 %
DC average = -4.08
Fluke measured DC average = -4.06
AC RMS = 4.30
Peak x .707 = 1.61
AC rectified average x 1.1 = 3.95
Fluke measured AC voltage = 4.01

Test 29:
Square Wave   4000 Hz
A = 2.8
B = -7.1
Duty Cycle = 72.7 %
DC average = 0.10
Fluke measured DC average = 0.00
AC RMS = 4.41
Peak x .707 = 1.91
AC rectified average x 1.1 = 4.36
Fluke measured AC voltage = 4.09

Test 30:
Square Wave   2000 Hz
A = 5.3
B = -4.8
Duty Cycle = 86.4 %
DC average = 3.93
Fluke measured DC average = 3.83
AC RMS = 3.46
Peak x .707 = 0.97
AC rectified average x 1.1 = 2.64
Fluke measured AC voltage = 2.54

Test 31:
Square Wave   500 Hz
A = 4.8
B = -5.2
Duty Cycle = 90.8 %
DC average = 3.88
Fluke measured DC average = 3.83
AC RMS = 2.89
Peak x .707 = 0.65
AC rectified average x 1.1 = 1.86
Fluke measured AC voltage = 1.89

Appendix 1: Average Voltage of a sine wave.

Refer to figures 1 through 4.
The peak-to-peak value for the waveform is (A-B).

The peak value (or amplitude) is then $\frac{(A-B)}{2}$.

The mid-point between A and B is then $\frac{(A+B)}{2}$.

The general formula for the waveform is then $v(t) = \frac{(A-B)}{2}\cos(t) + \frac{(A+B)}{2}$.

It is intuitive that the constant offset in the equation, which is also the mid-point between A and B, will be the average for the function. However, the formal derivation will still be performed.

The average is found as the integral of one period of the function divided by the period.

$$\text{Average Voltage} = \frac{1}{2\pi}\int_{0}^{2\pi} \frac{(A-B)}{2}\cos(t) + \frac{(A+B)}{2}$$

After integration, we have:

$$\text{Average Voltage} = \frac{1}{2\pi}\left. \frac{(A-B)}{2}\sin(t) + \frac{(A+B)}{2}t \right|_{0}^{2\pi}$$

Substituting the limits of the definite integral, we have:

$$\text{Average Voltage} = \frac{1}{2\pi}\left(\frac{(A-B)}{2}\sin(2\pi) + \frac{(A+B)}{2}2\pi - \frac{(A-B)}{2}\sin(0) - \frac{(A+B)}{2}0\right)$$

After some trigonometry, we have:

$$\text{Average Voltage} = \frac{1}{2\pi}\left(\frac{(A-B)}{2}0 + \frac{(A+B)}{2}2\pi - \frac{(A-B)}{2}0 - \frac{(A+B)}{2}0\right)$$

After some algebra, we have:

$$\textbf{Average Voltage} = \frac{(A+B)}{2}$$

Appendix 2: Total RMS Voltage of a sine wave.

Refer to figures 1 through 4.

The general formula for the waveform (from Appendix 1) is then $v(t) = \frac{(A-B)}{2}\cos(t) + \frac{(A+B)}{2}$.

Squaring v(t) gives:

$$v^2(t) = \frac{(A-B)^2}{4}\cos^2(t) + \frac{(A+B)^2}{4} + \frac{(A+B)(A-B)\cos(t)}{2}$$

Substituting into the formula for RMS voltage, we have:

$$\text{Total RMS Voltage} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \frac{(A-B)^2}{4}\cos^2(t) + \frac{(A+B)^2}{4} + \frac{(A+B)(A-B)\cos(t)}{2}}$$

After integration, we have:

$$\text{Total RMS Voltage} = \sqrt{\frac{1}{2\pi} \left. \frac{(A-B)^2}{4}\left(\frac{t}{2} + \frac{\sin(2t)}{4}\right) + \frac{(A+B)^2 t}{4} + \frac{(A+B)(A-B)\sin(t)}{2} \right|_0^{2\pi}}$$

After substituting the limits of the definite integral, we have:

$$= \sqrt{\frac{1}{2\pi}\left(\frac{(A-B)^2}{4}\left(\frac{2\pi}{2} + \frac{\sin(4\pi)}{4}\right) + \frac{(A+B)^2(2\pi)}{4} + \frac{(A+B)(A-B)\sin(2\pi)}{2} - \frac{(A-B)^2}{4}\left(\frac{0}{2} + \frac{\sin(0)}{4}\right) - \frac{(A+B)^2(0)}{4} - \frac{(A+B)(A-B)\sin(0)}{2}\right)}$$

After some trigonometry, we have:

$$= \sqrt{\frac{1}{2\pi}\left(\frac{(A-B)^2}{4}\left(\frac{2\pi}{2} + \frac{0}{4}\right) + \frac{(A+B)^2(2\pi)}{4} + \frac{(A+B)(A-B)0}{2} - \frac{(A-B)^2}{4}\left(\frac{0}{2} + \frac{0}{4}\right) - \frac{(A+B)^2(0)}{4} - \frac{(A+B)(A-B)0}{2}\right)}$$

After some algebra, we have:

Total RMS Voltage $= \sqrt{\frac{(A-B)^2}{8} + \frac{(A+B)^2}{4}}$

Appendix 3: AC RMS Voltage of a sine wave.

To get the AC component, we simply remove the DC average from the waveform. To do this we will use two new variables to indicate what the peaks of the waveform are after the Average Voltage is removed.
E = A-Average Voltage
F = B-Average Voltage

We can replace A and B with E and F in the formula for finding the Total RMS Voltage of a sine wave (Appendix 2).

AC RMS Voltage = $\sqrt{\frac{(E-F)^2}{8}+\frac{(E+F)^2}{4}}$

However, since the Average Voltage has been removed, we can simplify the formula.
The Average Voltage is $\frac{(A+B)}{2}$ (Appendix 1).

E = A - $\frac{(A+B)}{2}$

F = B - $\frac{(A+B)}{2}$

After some algebra, we find:

E = $\frac{(A-B)}{2}$

F = $\frac{(-A+B)}{2}$

From this we see that:
F = -E

Substituting –E for F, we get:

AC RMS Voltage = $\sqrt{\frac{(E+E)^2}{8}+\frac{(E-E)^2}{4}}$

After some algebra, we get the familiar formula for the RMS value of the AC component of a sine wave:

AC RMS Voltage = $\frac{E}{\sqrt{2}}$

Appendix 4: Rectified Average Voltage of a sine wave.

If the entire waveform is above the horizontal axis (i.e. $B \geq 0$), the Rectified Average Voltage will be the same as the Average Voltage.
If the entire waveform is below the horizontal axis (i.e. $A \leq 0$), the Rectified Average Voltage will be the negative of the Average Voltage.

For all other cases, v(t) will cross the horizontal axis. Please refer to fig. 3 and fig. 4.

(From Appendix 1.) $v(t) = \frac{(A-B)}{2}\cos(t) + \frac{(A+B)}{2}$

Due to the symmetry of a cosine wave, the first half (0 to π) of the period has the same average voltage as the entire period (0 to 2π). Thus, we need only consider the first half of the cosine wave.

Notice, on fig. 3, if we only look at the first half of the cosine wave, there is only one point that crosses the horizontal axis. That point is when $t = K\pi$, where K is a fraction such that $0 \leq K \leq 1$.

Since we are only considering the first half of the wave, we can break up the first half into two parts. The first part is when v(t) is positive ($0 \leq t \leq K\pi$), and the second is when v(t) is negative ($K\pi \leq t \leq \pi$). In order to rectify v(t), we could add the first part and the negative of the second part. For the first half of the wave, (which is all we need to consider), the rectified v(t) is as follows:

For $0 \leq t \leq K\pi$; $v_{rec}(t) = \frac{(A-B)}{2}\cos(t) + \frac{(A+B)}{2}$

For $K\pi \leq t \leq \pi$; $v_{rec}(t) = -\left(\frac{(A-B)}{2}\cos(t) + \frac{(A+B)}{2}\right)$

To get the Rectified Average Voltage, which is $\frac{1}{\pi}\int_0^{\pi} v_{rec}(t)$, we break the integral into its two parts:

$$\text{Rectified Average Voltage} = \frac{1}{\pi}\int_0^{K\pi} \frac{(A-B)}{2}\cos(t) + \frac{(A+B)}{2} - \frac{1}{\pi}\int_{K\pi}^{\pi} \frac{(A-B)}{2}\cos(t) + \frac{(A+B)}{2}$$

After integration we have:

$$\text{Rectified Average Voltage} = \frac{1}{\pi}\left|_0^{K\pi} \frac{(A-B)}{2}\sin(t) + \frac{(A+B)t}{2} - \frac{1}{\pi}\right|_{K\pi}^{\pi} \frac{(A-B)}{2}\sin(t) + \frac{(A+B)t}{2}$$

Substituting the limits of the definite integral we have:

$$= \frac{1}{\pi}\left(\frac{(A-B)}{2}\sin(K\pi) + \frac{(A+B)K\pi}{2} - \frac{(A-B)}{2}\sin(0) - \frac{(A+B)0}{2}\right) - \frac{1}{\pi}\left(\frac{(A-B)}{2}\sin(\pi) + \frac{(A+B)\pi}{2} - \frac{(A-B)}{2}\sin(K\pi) - \frac{(A+B)K\pi}{2}\right)$$

After some trigonometry, we have:

$$= \frac{1}{\pi}\left(\frac{(A-B)}{2}\sin(K\pi) + \frac{(A+B)K\pi}{2} - \frac{(A+B)0}{2}\right) - \frac{1}{\pi}\left(\frac{(A+B)\pi}{2} - \frac{(A-B)}{2}\sin(K\pi) - \frac{(A+B)K\pi}{2}\right)$$

After some algebra, we have:

Rectified Average Voltage $= \frac{(A-B)}{\pi}\sin(K\pi) + K(A+B) - \frac{(A+B)}{2}$

Since we know that v(t) is 0 when t = $K\pi$, we can derive a formula for K. Substituting $K\pi$ for t in the equation v(t) = 0, we have:

$$\frac{(A-B)}{2}\cos(K\pi)+\frac{(A+B)}{2}=0$$

$$\frac{(A-B)}{2}\cos(K\pi)=-(A+B)$$

$$\cos(K\pi)=-\frac{(A+B)}{(A-B)}$$

$$\cos(K\pi)=\frac{(B+A)}{(B-A)}$$

$$K\pi=\mathrm{ar}\cos\left(\frac{(B+A)}{(B-A)}\right)$$

$$\mathbf{K}=\frac{1}{\pi}\mathrm{ar}\cos\left(\frac{B+A}{B-A}\right)$$

Appendix 5: AC Rectified Average Voltage of a sine wave.

As in the case of finding the AC RMS value (Appendix 3), we can simply remove the DC from the waveform and use the formula for Rectified Average Voltage to find the AC Rectified Average Voltage. As before, E and F will be the peak values of the waveform after the DC Average has been removed. Therefore, as before, E = -F. Similarly, F = -E

Substituting E and –E for A and B in the equation for Rectified Average Voltage (Appendix 4), we have:

AC Rectified Average Voltage = $\frac{(E+E)}{\pi}\sin(K\pi)+K(E-E)-\frac{(E-E)}{2}$

After algebra we have:

AC Rectified Average Voltage = $\frac{2E}{\pi}\sin(K\pi)$

The formula for K is found similarly:

$$K=\frac{1}{\pi}\mathrm{ar}\cos\left(\frac{-E+E}{-E-E}\right)$$

After algebra, we have:

$$K=\frac{1}{\pi}\mathrm{ar}\cos(0)$$

After some trigonometry, we have:

$$K=\frac{1}{\pi}\left(\frac{\pi}{2}\right)$$

We find K is simply a constant: K = $\frac{1}{2}$

If we substitute K into the formula for AC Rectified Average Voltage, we have:

AC Rectified Average Voltage = $\frac{2E}{\pi}\sin\left(\frac{\pi}{2}\right)$

After some trigonometry we have:

AC Rectified Average Voltage = $\frac{2E}{\pi}$

Appendix 6: Average Voltage of a sawtooth wave.

Refer to figures 5 through 8.
The voltage function v(t) for one period of the waveform is simply a line with slope.
The general formula for a line with slope is y = mx + b where m is slope and b is the y-intercept.

For v(t), $\frac{(A-B)}{T}$ is the slope, and B is the y-intercept.

Thus we have: $v(t) = \frac{(A-B)t}{T} + B$.

It is intuitive that the mid-point between A and B, will be the average for the function. However, the formal derivation will still be performed.

The average is found as the integral of the function divided by one period of the function.

$$\text{Average Voltage} = \frac{1}{T}\int_0^T \frac{(A-B)t}{T} + B$$

After integration, we have:

$$\text{Average Voltage} = \frac{1}{T}\left|_0^T \frac{(A-B)t^2}{2T} + Bt\right.$$

Substituting the limits of the definite integral, we have:

$$\text{Average Voltage} = \frac{1}{T}\left(\frac{(A-B)T^2}{2T} + BT - \frac{(A-B)0^2}{2T} - B(0)\right)$$

After some algebra, we have:

$$\textbf{Average Voltage} = \frac{(A+B)}{2}$$

Appendix 7: Total RMS Voltage of a sawtooth wave.

Refer to figures 5 through 8.

The general formula for the waveform (from Appendix 6) is: $v(t) = \frac{(A-B)t}{T} + B$.

Squaring v(t) gives:

$$v^2(t) = \frac{(A-B)^2 t^2}{T^2} + \frac{2B(A-B)t}{T} + B^2$$

Substituting into the formula for RMS voltage, we have:

$$\text{Total RMS Voltage} = \sqrt{\frac{1}{T}\int_0^T \frac{(A-B)^2 t^2}{T^2} + \frac{2B(A-B)t}{T} + B^2}$$

After integration, we have:

$$\text{Total RMS Voltage} = \sqrt{\frac{1}{T}\left| \frac{(A-B)^2 t^3}{3T^2} + \frac{B(A-B)t^2}{T} + B^2 t \right._0^T}$$

After substituting the limits of the definite integral, we have:

$$\text{Total RMS Voltage} = \sqrt{\frac{1}{T}\left(\frac{(A-B)^2 T^3}{3T^2} + \frac{B(A-B)T^2}{T} + B^2 T \right)}$$

After some algebra, we have:

$$\textbf{Total RMS Voltage} = \sqrt{\frac{A^2 + AB + B^2}{3}}$$

Appendix 8: AC RMS Voltage of a sawtooth wave.

To get the AC component, we simply remove the DC average from the waveform. To do this we will use two new variables to indicate what the peaks of the waveform are after the Average Voltage is removed.

E = A-Average Voltage
F = B-Average Voltage

We can replace A and B with E and F in the formula for finding the Total RMS Voltage of a sawtooth wave (Appendix 7).

$$\text{AC RMS Voltage} = \sqrt{\frac{E^2 + EF + F^2}{3}}$$

However, since the Average Voltage has been removed, we can simplify the formula.

The Average Voltage is $\frac{(A+B)}{2}$ (Appendix 6).

$$E = A - \frac{(A+B)}{2}$$

$$F = B - \frac{(A+B)}{2}$$

After some algebra, we find:

$$E = \frac{(A-B)}{2}$$

$$F = \frac{(-A+B)}{2}$$

From this we see that:
F = -E

Substituting –E for F, we get:

$$\text{AC RMS Voltage} = \sqrt{\frac{E^2 + E(-E) + (-E)^2}{3}}$$

After some algebra, we have the formula for the RMS value of the AC component of a sawtooth wave:

AC RMS Voltage = $\frac{E}{\sqrt{3}}$

Appendix 9: Rectified Average Voltage of a sawtooth wave.

If the entire waveform is above the horizontal axis (i.e. $B \geq 0$), the Rectified Average Voltage will be the same as the Average Voltage.
If the entire waveform is below the horizontal axis (i.e. $A \leq 0$), the Rectified Average Voltage will be the negative of the Average Voltage.

For all other cases, v(t) will cross the horizontal axis. Please refer to fig. 7 and fig. 8.

(From Appendix 6.) $v(t) = \frac{(A-B)t}{T} + B$

We can break up the function into two parts. The first part is when v(t) is negative ($0 \leq t \leq GT$), and the second is when v(t) is positive ($GT \leq t \leq T$). In order to rectify v(t), we could add the second part and the negative of the first part. Thus, the rectified v(t) is as follows:

For $0 \leq t \leq GT$; $v_{rec}(t) = -\left(\frac{(A-B)t}{T} + B\right)$

For $GT \leq t \leq T$; $v_{rec}(t) = \frac{(A-B)t}{T} + B$

To get the Rectified Average Voltage, which is $\frac{1}{T}\int_0^T v_{rec}(t)$, we break the integral into its two parts:

$$\text{Rectified Average Voltage} = \frac{-1}{T}\int_0^{GT} \frac{(A-B)t}{T} + B + \frac{1}{T}\int_{GT}^{T} \frac{(A-B)t}{T} + B$$

After integration we have:

$$\text{Rectified Average Voltage} = \frac{-1}{T}\Bigg|_0^{GT} \frac{(A-B)t^2}{2T} + Bt + \frac{1}{T}\Bigg|_{GT}^{T} \frac{(A-B)t^2}{2T} + Bt$$

Substituting the limits of the definite integral we have:

$$= \frac{-1}{T}\left(\frac{(A-B)G^2T^2}{2T} + BGT - \frac{(A-B)0}{2T} - B(0)\right) + \frac{1}{T}\left(\frac{(A-B)T^2}{2T} + BT - \frac{(A-B)G^2T^2}{2T} - BGT\right)$$

After some algebra, we have:

$$\text{Rectified Average Voltage} = -(A-B)G^2 - 2BG + \frac{(A-B)}{2} + B$$

Since we know v(t) = 0 when t = GT, we can derive a formula for G. Substituting GT for t in the equation v(t) = 0, we have:

$$\frac{(A-B)GT}{T} + B = 0$$

After some algebra, we have:

$$G = \frac{-B}{A-B}$$

Substituting the formula for G into the equation for Rectified Average Voltage, we have:

$$\text{Rectified Average Voltage} = \frac{-(A-B)B^2}{(A-B)^2} + \frac{2B^2}{(A-B)} + \frac{(A-B)}{2} + B$$

After some algebra, we have:

Rectified Average Voltage $= \dfrac{A^2 + B^2}{2A - 2B}$

Appendix 10: AC Rectified Average Voltage of a sawtooth wave.

As in the case of finding the AC RMS value (Appendix 8), we can simply remove the DC from the waveform and use the formula for Rectified Average Voltage to find the AC Rectified Average Voltage. As before, E and F will be the peak values of the waveform after the DC Average has been removed. Therefore, as before, E = -F. Similarly, F = -E

Substituting E and –E for A and B in the equation for Rectified Average Voltage (Appendix 9), we have:

$$\text{AC Rectified Average Voltage} = \frac{E^2 + E^2}{2E + 2E}$$

After algebra we have:

AC Rectified Average Voltage $= \dfrac{E}{2}$

Appendix 11: Average Voltage of a square wave.

Refer to figures 17 through 20.

The voltage v(t), for one period of the waveform, is simply made up of two discreet, horizontal lines with no slope. Strictly speaking, the idealized waveform described here is not a function. It is a discontinuous waveform. However, for our purposes, the waveform can be described as two separate functions combined.

D is the duty cycle. It represents the fraction of the total period T, during which the waveform exists at voltage A. Therefore, D must be $0 \le D \le 1$. v(t) is described as follows:

For $0 < t < DT$; $v(t) = A$
For $DT < t < T$; $v(t) = B$

The mid-point between A and B, is not necessarily the average for the function in this case due to the effects of the duty cycle. The average is found as the integral of one period of the waveform divided by the period.

$$\text{Average Voltage} = \frac{1}{T}\int_0^{DT} A + \frac{1}{T}\int_{DT}^{T} B$$

After integration, we have:

$$\text{Average Voltage} = \frac{1}{T}\Big|_0^{DT} At + \frac{1}{T}\Big|_{DT}^{T} Bt$$

Substituting the limits of the definite integral, we have:

$$\text{Average Voltage} = \frac{1}{T}\left(ADT - A(0)\right) + \frac{1}{T}\left(BT - BDT\right)$$

After some algebra, we have:

Average Voltage = $AD + B - BD$

Note: if D = 1/2, then Average Voltage will be $\frac{A + B}{2}$.

Appendix 12: Total RMS Voltage of a square wave.

Refer to figures 17 through 20.
The general formula of the waveform (from Appendix 11) is:
For $0 < t < DT$; $v(t) = A$
For $DT < t < T$; $v(t) = B$

Squaring v(t) gives:
For $0 < t < DT$; $v^2(t) = A^2$
For $DT < t < T$; $v^2(t) = B^2$

Substituting into the formula for RMS voltage, we have:

$$\text{Total RMS Voltage} = \sqrt{\frac{1}{T}\int_0^{DT} A^2 + \frac{1}{T}\int_{DT}^{T} B^2}$$

After integration, we have:

$$\text{Total RMS Voltage} = \sqrt{\frac{1}{T}\Big|_0^{DT} A^2 t + \frac{1}{T}\Big|_{DT}^{T} B^2 t}$$

After substituting the limits of the definite integral, we have:

$$\text{Total RMS Voltage} = \sqrt{\frac{1}{T}\left(A^2 DT - A^2(0)\right) + \frac{1}{T}\left(B^2 T - B^2 DT\right)}$$

After some algebra, we have:

Total RMS Voltage = $\sqrt{A^2 D + B^2 - B^2 D}$

Appendix 13: AC RMS Voltage of a square wave.

To get the AC component, we simply remove the DC average from the waveform. To do this we will use two new variables to indicate what the peaks of the waveform are after the Average Voltage is removed.

E = A-Average Voltage
F = B-Average Voltage

We can replace A and B with E and F in the formula for finding the Total RMS Voltage of a square wave (Appendix 12).

AC RMS Voltage = $\sqrt{E^2 D + F^2 - F^2 D}$

However, if D is 1/2, we can simplify the formula.

The Average Voltage is $AD + B - BD$ (Appendix 11).

$E = A - (AD + B - BD)$
$F = B - (AD + B - BD)$

After some algebra, we find:

$E = \dfrac{A - B}{2}$

$F = \dfrac{-A + B}{2}$

From this we see that:
F = -E (when D = 1/2)

Substituting –E for F, we get:
AC RMS Voltage = $\sqrt{E^2 D + E^2 - E^2 D}$

After some algebra, we have the formula for the RMS value of the AC component of a square wave if the duty cycle is 1/2:

AC RMS Voltage = E (Only when D =1/2)

Appendix 14: Rectified Average Voltage of a square wave.

If the entire waveform is above the horizontal axis (i.e. $B \geq 0$), the Rectified Average Voltage will be the same as the Average Voltage.
If the entire waveform is below the horizontal axis (i.e. $A \leq 0$), the Rectified Average Voltage will be the negative of the Average Voltage.

For all other cases, v(t) will cross the horizontal axis. Please refer to fig. 19 and fig. 20.

(From Appendix 11):
For $0 < t < DT$; $v(t) = A$
For $DT < t < T$; $v(t) = B$

Rectifying this waveform is simple. It is already in two parts. The first part is when v(t) is positive ($0 < t < DT$), and the second is when v(t) is negative ($DT < t < T$). In order to rectify v(t), we could add the first part and the negative of the second part. Thus, the rectified v(t) is as follows:

For $0 < t < DT$; $v_{rec}(t) = A$
For $DT < t < T$; $v_{rec}(t) = -B$

To get the Rectified Average Voltage, which is $\frac{1}{T}\int_0^T v_{rec}(t)$, we break the integral into its two parts:

$$\text{Rectified Average Voltage} = \frac{1}{T}\int_0^{DT} A + \frac{1}{T}\int_{DT}^{T} -B$$

After integration we have:

$$\text{Rectified Average Voltage} = \frac{1}{T}\Big|_0^{DT} At + \frac{1}{T}\Big|_{DT}^{T} -Bt$$

Substituting the limits of the definite integral we have:

$$= \frac{1}{T}\left(ADT - A(0)\right) + \frac{1}{T}\left(-BT + BDT\right)$$

After some algebra, we have:
Rectified Average Voltage $= AD - B + BD$

However, if D is 1/2, we can simplify the formula.

After some algebra, we have:

$$\text{Rectified Average Voltage} = \frac{A - B}{2} \quad \text{(Only when D = 1/2)}$$

Appendix 15: AC Rectified Average Voltage of a square wave.

As in the case of finding the AC RMS value (Appendix 13), we can simply remove the DC from the waveform and use the formula for Rectified Average Voltage, to find the AC Rectified Average Voltage.

As before, E and F will be the peak values of the waveform after the DC Average has been removed.

Substituting E and F for A and B in the equation for Rectified Average Voltage (Appendix 14)we have:

AC Rectified Average Voltage = $ED - F + FD$

However, if D is 1/2, we can simplify the formula.

From Appendix 13 we know:

$F = -E$ (when $D = 1/2$)

Substituting –E for F, we get:

Rectified Average Voltage = $ED + E - ED$

After some algebra, we have the formula for the AC Rectified Average Voltage of a square wave if the duty cycle is 1/2:

AC Rectified Average Voltage = E (Only when $D = 1/2$)

Appendix 16: Qbasic voltage calculator.

This program calculates the different types of voltages for a given waveform. Given the type of waveform, the peak voltages (A and B), and the duty cycle (if applicable), the program calculates the Average Voltage, Total RMS Voltage, AC RMS Voltage, Rectified Average Voltage, and the AC Rectified Average Voltage. To use it, copy and paste the code into a raw-text editor such as MS notepad. There must be no lines above the first line of code. Save the file as raw text (no formatting) and rename it with a .bas extension instead of .txt. It should then open as a Qbasic file if you have Qbasic on your computer. If not, then you may have to search the web to get a free copy of Qbasic.

The code starts here:

```
DECLARE SUB sinewave ()
DECLARE SUB squarewave ()
DECLARE SUB sawtoothwave ()
DIM SHARED waveType      AS INTEGER
DIM SHARED outputFile    AS INTEGER
DIM SHARED pi  AS SINGLE
pi = 3.141593
DIM SHARED A   AS SINGLE
DIM SHARED B   AS SINGLE
DIM SHARED D   AS SINGLE
DIM SHARED E   AS SINGLE
DIM SHARED F   AS SINGLE
DIM SHARED K   AS SINGLE
DIM SHARED X   AS SINGLE
DIM SHARED arccosX      AS SINGLE
DIM SHARED average       AS SINGLE
DIM SHARED totalRMS     AS SINGLE
DIM SHARED acRMS        AS SINGLE
DIM SHARED rectAve         AS SINGLE
DIM SHARED acRectAve     AS SINGLE
DIM SHARED acRctAveCF   AS SINGLE
SCREEN 12
CLS
10
PRINT "Enter the type of waveform you want to analyze."
PRINT "1 for sine wave, 2 for square wave, "
INPUT "3 for sawtooth/triangle wave, 4 to end program."; waveType
IF waveType = 4 THEN SYSTEM
PRINT " "
CLOSE
INPUT "Enter 1 if you want to save the output to a file called RMSOUT.TXT"; outputFile
PRINT " "
IF outputFile = 1 THEN
    OPEN "RMSout.txt" FOR APPEND AS #1
END IF
IF waveType = 1 THEN
                CALL sinewave
ELSEIF waveType = 2 THEN
                CALL squarewave
ELSEIF waveType = 3 THEN
                CALL sawtoothwave
ELSE GOTO 10
END IF
PRINT "Average Voltage ="; average
PRINT "Total RMS Voltage ="; totalRMS
PRINT "AC RMS Voltage ="; acRMS
PRINT "Rectified Average Voltage ="; rectAve
```

```
PRINT "AC Rectified Average Voltage ="; acRectAve
PRINT "AC Rectified Average times 1.110721 (AC voltmeter reading) ="; acRctAveCF
PRINT " "
PRINT " "
IF outputFile = 1 THEN
    PRINT #1, "Average Voltage ="; average
    PRINT #1, "Total RMS Voltage ="; totalRMS
    PRINT #1, "AC RMS Voltage ="; acRMS
    PRINT #1, "Rectified Average Voltage ="; rectAve
    PRINT #1, "AC Rectified Average Voltage ="; acRectAve
    PRINT #1, "AC Rectified Average times 1.110721 (AC voltmeter reading) =";
acRctAveCF
    PRINT #1, " "
    PRINT #1, " "
END IF
CLOSE
GOTO 10

SUB sawtoothwave
100
INPUT "enter A, (the upper peak voltage)"; A
INPUT "enter B, (the lower peak voltage)"; B
PRINT " "
IF A < B THEN
   PRINT "A must be greater than or equal to B."
   GOTO 100
END IF
average = (A + B) / 2
totalRMS = SQR((A ^ 2 + A * B + B ^ 2) / 3)
E = A - average
F = B - average
acRMS = E / SQR(3)
IF B > 0 OR B = 0 THEN
                rectAve = average
ELSEIF A < 0 OR A = 0 THEN
                rectAve = -average
ELSE
    rectAve = (A ^ 2 + B ^ 2) / (2 * A - 2 * B)
END IF
acRectAve = E / 2
acRctAveCF = acRectAve * 1.110721#
PRINT "Sawtooth/Triangle Wave"
PRINT "Upper Peak Voltage A ="; A
PRINT "Lower Peak Voltage B ="; B
IF outputFile = 1 THEN
    PRINT #1, "Sawtooth/Triangle Wave"
    PRINT #1, "Upper Peak Voltage A ="; A
    PRINT #1, "Lower Peak Voltage B ="; B
END IF
END SUB

SUB sinewave
200
INPUT "enter A, (the upper peak voltage)"; A
INPUT "enter B, (the lower peak voltage)"; B
PRINT " "
IF A < B THEN
   PRINT "A must be greater than or equal to B."
   GOTO 200
END IF
average = (A + B) / 2
```

```
totalRMS = SQR((((A - B) ^ 2) / 8) + (((A + B) ^ 2) / 4))
E = A - average
F = B - average
acRMS = E / SQR(2)
IF B > 0 OR B = 0 THEN
                rectAve = average
ELSEIF A < 0 OR A = 0 THEN
                rectAve = -average
ELSE
    X = (B + A) / (B - A)
    IF X = 1 THEN
             arccosX = 0
        ELSE
             arccosX = ATN(-X / SQR(1 - X ^ 2)) + pi / 2
    END IF
    K = (1 / pi) * arccosX
    rectAve = (((A - B) / pi) * SIN(K * pi)) + K * (A + B) - ((A + B) / 2)
END IF
acRectAve = (2 * E) / pi
acRctAveCF = acRectAve * 1.110721#
PRINT "Sine Wave"
PRINT "Upper Peak Voltage A ="; A
PRINT "Lower Peak Voltage B ="; B
IF outputFile = 1 THEN
    PRINT #1, "Sine Wave"
    PRINT #1, "Upper Peak Voltage A ="; A
    PRINT #1, "Lower Peak Voltage B ="; B
END IF
END SUB

SUB squarewave
300
INPUT "enter A, (the upper peak voltage)"; A
INPUT "enter B, (the lower peak voltage)"; B
PRINT " "
IF A < B THEN
   PRINT "A must be greater than or equal to B."
   GOTO 300
END IF
310
PRINT "enter D, (the duty cycle of the square wave)"
PRINT "The duty cycle D is the fraction of a period"
PRINT "during which the wave remains at A."
INPUT "Therefore D can be from 0 to 1"; D
PRINT " "
IF D > 1 GOTO 310
IF D < 0 GOTO 310
average = A * D + B - B * D
totalRMS = SQR(((A ^ 2) * D) + (B ^ 2) - ((B ^ 2) * D))
E = A - average
F = B - average
acRMS = SQR(((E ^ 2) * D) + (F ^ 2) - ((F ^ 2) * D))
IF B > 0 OR B = 0 THEN
                rectAve = average
ELSEIF A < 0 OR A = 0 THEN
                rectAve = -average
ELSE
       rectAve = A * D - B + B * D
END IF
acRectAve = E * D - F + F * D
acRctAveCF = acRectAve * 1.110721#
```

```
PRINT "Square Wave"
PRINT "Upper Peak Voltage A ="; A
PRINT "Lower Peak Voltage B ="; B
PRINT "Duty Cycle is"; D * 100; "%"
IF outputFile = 1 THEN
    PRINT #1, "Square Wave"
    PRINT #1, "Upper Peak Voltage A ="; A
    PRINT #1, "Lower Peak Voltage B ="; B
    PRINT #1, "Duty Cycle is"; D * 100; "%"
END IF
END SUB
```